

California Real Estate Finance
Chapter 15
Mathematics of Real Estate Finance

Chapter Purpose

This final chapter is designed to put the mathematics in one place. Much of this material has probably already been presented with prior chapters or covered in other courses. However, it is important to review these math facts in order to understand how real estate finance impacts and investment's profitability.

All finance math includes an examination of payments, income and expenses continuing for relatively long periods of time. This forces the students to examine the time value of money and its implications on investment decision making. Each purchaser who borrows money will be concerned on how the monthly payments are derived and how the charges and credits are prorated and allocated in the closing statement. Every real estate investor must master the math involved in measuring profitability. Many students will find discounting to be an integral part of finance knowledge.

Suggestions to the Instructor

If you have already covered the material in this chapter, you will probably skip it. It is placed here as an easy reference for the reader of the text. The only thing you might want to consider adding to the presentation of any of the material from this chapter is a discussion on probability theory, which is applicable in current realty appraising. We hope you found this Total Instructor Resource Package helpful in your course work and that you will continue to use the text for future classes in Real Estate Finance.

Learning Objectives

Upon completion of this chapter, the student should be able to:

1. define the different types of interest: simple, add-on and compound;
2. differentiate between nominal and effective rates of interest;
3. derive the compound worth of an annuity;
4. understand and explain the concept of the time value of money;
5. evaluate the present worth of a stream of income;
6. develop an amortization schedule for any principal amount at any rate of interest for a specified amount of time;
7. prorate all items in a closing statement and allocate charges and credits to the appropriate parties;

8. measure the profitability of a real estate investment, including the project's break-even amount and its bottom-line, after-tax, return on monies invested.

Presentation Outline

- I. Interest
 - A. Simple interest
 1. "Rent" paid for the use of money
 2. "Rent" paid only on the amount of principal still owed
 3. Interest ceases on principal repaid
 4. Derived from $I=PRT$
 - B. Add-on interest
 1. "Rent" paid on the entire amount of principal for the entire period
 2. Disregards any principal repaid
 3. Acts to almost double contract rate
 4. Derived from $AIR=2IC/P (n+1)$
 - C. Rates of interest
 1. Nominal rate of interest is contracted rate
 2. Effective rate of interest includes additional costs, charges and discounts
 - D. Compound interest
 1. Interest paid on interest earned
 2. More frequent compounding raises yield
 - a. annual compounding
 - b. monthly
 - c. daily
 3. Derived form $CS=BD (1+i)^n$
 - E. Compound worth of an annuity
 1. Compounds the interest on a series of deposits or receipts
 2. Derived from $CS=RD (1+I)^{n-1} + (1+I)^{n-2} \dots (1+i)^{n-n}$
- II. Time Value of Money
 - A. Present worth of money
 1. Money received in the future worth less today
 2. Discount is earnings lost during waiting period
 - a. called the "opportunity cost"
 - b. a function of time and required yield
 - B. Present worth of a dollar
 1. The reciprocal of the compound formula
 2. \$1 at 6% is worth \$1.06 at the end of the year
 3. To wait a year for \$1 makes it worth \$0.94 today at 6% interest
 4. \$0.06 is the opportunity cost or "discount"
 5. Derived from $PW = A[1/(1+i)^n]$

- C. Present worth of an annuity
 - 1. The reciprocal of the compound formula
 - 2. \$1 to be received at the end of each year for three years at 6% is worth \$2.67 today
 - 3. Derived: $PWA = RA[1/(1+i)^n + 1/(1+i)^{n-1} + 1/(1+i)^{n-2}]$
 - 4. Can be applied to a regular annuity
 - 5. Can be applied to an irregular annuity

III. Payment Schedules

A. Amortization

- 1. The systematic repayment of a debt
 - a. annual payments
 - b. monthly payments
 - c. loan constants
- 2. Distribution of principal and interest
 - a. interest credited first
 - b. balance of payment to principal
 - c. principal portion deducted from loan balance
 - d. reduces interest and raises principal portions in next payment
 - e. gradually repays loan over time
- 3. Total interest costs
 - a. the total interest amount paid on a \$100,000 loan at 8% for 30 years is \$164,301.20
 - b. to save this interest, pay loan off sooner
 - c. but then you lose earnings on your savings
 - d. every investment has an opportunity cost

B. Term loans

- 1. Any loan that is designed to be paid in full prior to its normal amortization period
 - a. 30-year loan due in full in 10 years
 - (1) stop date
 - (2) balloon payment
- 2. Bi-weekly loan
 - a. 26 payments
 - b. includes one extra month's payment per year

C. Impound funds (escrow accounts)

- 1. Advance collections for future payments
 - a. property taxes
 - b. insurance premiums
 - c. assessments
 - d. mortgage insurance premiums

IV. Closing Statements Prorations

A. Interest

1. Usually paid in arrears
2. Assumed loan
 - a. interest prorated to date of closing
 - b. charged to seller and credited to buyer for next payment
3. New loan
 - a. interest prorated from date of closing to day first payment is due
 - b. amount is charged to buyer-borrower

B. Property taxes

1. Imposed on fiscal basis: July 1 to June 30
2. First half due November 1, second half February 1
3. Full year may be paid at any time
4. Usually paid in arrears
5. Proration is from date last paid to closing date
6. Charged to seller, credited to buyer

C. Insurance premiums

1. Assumed policy
 - a. always paid in advance
 - b. proration from policy renewal date to closing
 - c. credited to seller, charged to buyer
2. New policy
 - a. always paid in advance
 - b. total premium charged to buyer-borrower

D. Impound funds

1. Amount for taxes and insurance
2. Placed into collection escrow to accumulate for payment when due
 - a. number of months required to be paid into escrow depends on comparing closing date with dates of when taxes and insurance premiums are due
 - b. impound amount plus future months' payments must total enough to meet renewal amounts
3. Assumed loan
 - a. balance of impound account
 - b. credited to seller, charged to buyer
4. New loan
 - a. required number of months amount
 - b. charged to buyer-borrower

E. Other items

1. Assessments
2. Recreation and homeowners' association fees
3. Rents and rental deposits
4. Utilities usually handled outside of closing
5. Transfer tax usually charged to seller
 - a. rate is \$1.10 per thousand of equity involved
 - b. no tax on assumed loan amounts

- V. Measuring Profitability
 - A. Breakeven analysis
 - 1. Fixed costs
 - a. loan payments
 - b. taxes and insurance premiums
 - c. utilities and maintenance (some variable)
 - d. management
 - 2. Variable costs
 - a. utilities and maintenance (partially fixed)
 - b. bookkeeping and advertising
 - 3. Derived from $BE = FC / 1 - VCR$
 - B. Return on investment
 - 1. Lender's profitability calculations
 - a. project's net annual income divided by
 - b. market capitalization rate
 - c. equals value of project
 - d. derived from $V = I/R$
 - 2. Investor's profitability calculations (capitalization rate)
 - a. built-up rate
 - b. weighted rate
 - c. allocate weights to portions of equity and financing
 - (1) 75% interest rate on senior loan
 - (2) 15% interest rate on junior loan
 - (3) 10% interest rate on equity portion
 - 3. Net present worth method
 - a. applies present worth of money theory to project's cash flow
 - b. discounts each year's net cash flow and the residual when the property will be sold
 - c. discount rate is determined by investor
- VI. Discounting Trust Deeds and Mortgages
 - A. Points
 - 1. Loan originators charge points
 - a. to offset operational costs
 - b. to offset discounts of secondary market
 - 2. One point equals 1% of loan amount
 - 3. One point is equated with raising the effective interest rate on the loan by $\frac{1}{8}\%$
 - a. a one point discount on an 8% loan will raise its effective rate to $8\frac{1}{8}\%$
 - b. a two point discount to $8\frac{1}{4}\%$

B. Discounts

1. Rule of thumb
 - a. take difference of the nominal rate and the expected rate
 - b. multiply answer by the number of years the loan has left
2. Discounted cash flow
 - a. discount income stream by required rate
 - b. discount balloon payment by same rate